Exponential Growth

Exponential Relations: \( y = A(b)^x \)  
\( A = \) initial amount \( \quad b = \) base  
\( x = \) growth intervals

Exponential growth when \( b > 1 \)

Some applications of exponential growth are:

- Bacteria growth (\( b = 2 \) usually)
- Population growth (\( b = 1 + \text{pop. Growth %} \))
- Price Increases (\( b = 1 + \text{price increase %} \))

NBA Salaries:
NBA salaries can be modeled by \( S = 4.2(1.146)^n \) where \( S \) is the salary in millions, and \( n \) is the number of years since 2000.

a) What type of equation is this?

b) What were players’ salaries in 2000?

c) What are players’ salaries in 2000? \( n = \)

b) What is the annual growth rate (in percent)?

e) When will players reach 20 million a year?

Population Growth
The population of a small country grows 1.5% every year. The country’s population is now 400,000.

a) Give an equation for the population in terms of \( n \) (the number of years).

b) When will the population reach its carrying capacity (maximum population)? Assume that a country can only sustain 14 million.

Information from a table of values:

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>400</td>
<td>162</td>
</tr>
<tr>
<td>2013</td>
<td>480</td>
<td>120</td>
</tr>
<tr>
<td>2014</td>
<td>576</td>
<td>1.2</td>
</tr>
<tr>
<td>2015</td>
<td>691</td>
<td>1.2</td>
</tr>
<tr>
<td>2016</td>
<td>829</td>
<td>1.2</td>
</tr>
<tr>
<td>2017</td>
<td>995</td>
<td>1.2</td>
</tr>
</tbody>
</table>

A biologist tracks the population of a new species of frog over several years. From the table of values, determine an equation that models the frog’s population growth, and determine the number of years before the population triples.

\[ P = 400(1.2)^n \text{ Tr. Pop. 1200 About 6 yrs} \]
Investment:

Suppose you invest $1000 in a mutual fund that earns 2% interest per month.

a) Give an equation for the value of the fund in terms of the number of months.

\[ V = 1000 \left(1 + 0.02 \right)^n \]

b) How much will the fund have after 5 months?

\[ V = 1000 \left(1 + 0.02 \right)^5 \approx 1104.08 \]

c) How much will the fund have after one year?

\[ V = 1000 \left(1 + 0.02 \right)^{12} \approx 1268.24 \]

Practice:

1. Truong studied a pond where frogs lay eggs. He found the number of tadpoles increased by a factor of 2.43 per day. The number of tadpoles, \( T \), can be modelled by the relation \( T = 265(2.43)^t \), where \( t \) is the time in days.
   a) How many tadpoles were present at the start of Truong's study?

b) How many tadpoles were present after two days?

c) How many tadpoles were present after one week?
2. Cells in a culture are doubling every day. The number of cells in the culture, N, can be estimated using the formula $N = 1000(2)^d$, where $d$ is the number of days.
   a) How many cells does the culture begin with?
   
   b) How many cells would there be after 2 days?
   
   c) How many days will it take to reach 16,000 cells?

3. The world’s population in 1980 was about 4.5 billion. Suppose the population increased at a rate of 2% per year since then.
   
   a) Write an exponential relation that models the problem. Remember $b = 1 + \%$ for growth. Explain what each variable represents.
   
   b) What will be the world’s population in 2015?

4. The population of a city grows at a rate of 5% per year. The population in 1990 was 400,000. What would be the predicted current population? In what year would we predict the population to reach 1,000,000?