Quadratic Relations: Vertex to Standard Form

Recall Vertex Form: \( y = a(x - h)^2 + k \)  What does the ‘a’ tell us? 
What is (h,k)?

Now Standard Form: \( y = ax^2 + bx + c \)  What does the ‘a’ tell us? 
What does the ‘c’ tell us?  +“b” does not give us any direct information

Example 1: Given \( y = (x - 6)(x + 1) \)
   a) Expand and place the equation in standard form.
      \[ y = x^2 - 5x - 6 \]

   b) State the y–intercept and the step pattern.

Example 2: Given \( y = (x - 2)^2 + 1 \)
   a) Sketch the graph on the grid below

   b) In what form is the equation? 

   c) Fill in the table below using the equation \( y = x^2 - 4x + 5 \) and then sketch the graph by plotting the coordinates.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
</tbody>
</table>

   d) What do you notice about the two graphs?

   e) Expand and simplify:
      \[ y = (x - 2)^2 + 1 \]
      \[ y = x^2 - 4x + 5 \]
      Same, quadratic equation in a different form

   f) What do you notice about the two equations?
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**Example:**
For each set of parabolas, use Desmos to graph each and determine the one that doesn't belong.

a) \( y = 2(x - 1)^2 - 8 \)  
\[ y = 2x^2 - 4x - 6 \]
\[ y = 2(x - 1)^2 - 8 = 2(x - 1)(x - 2) \]
\[ y = 2(x - 3)(x + 1) \]

b) \( y = 4(x - 0)^2 + 16 \)  
\[ y = 4x^2 + 0x + 16 \]
\[ y = 4(x - 0)^2 + 16 = 4(x - 2)^2 \]

(c) \( y = -3(x - 2)^2 \)  
\[ y = -3x^2 + 12x - 12 \]
\[ y = 3x^2 - 12x + 12 \]

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**Key Concepts**

- A quadratic relation can be written in vertex form, \( y = a(x-h)^2 + k \), or in standard form, \( y = ax^2 + bx + c \).
- Expand and simplify the vertex form to write the quadratic relation in standard form.
- Given a quadratic relation in vertex form, \( y = a(x-h)^2 + k \), the coordinates of the vertex are \((h, k)\).
- Given a quadratic relation in standard form, \( y = ax^2 + bx + c \), the \( y \)-intercept is \( c \).

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The strategy for a long distance rollerblading is to find the best speed that can be maintained for a long time. A racer’s performance can be modeled by the quadratic relation \( d = -2(s-6)^2 + 72 \), where \( d \) is the racer’s maximum distance travelled, in km, at a speed, \( v \), in metres per second.

a) What are the coordinates of the vertex?

\( (6, 72) \)

b) What is maximum distance?

72 m

c) What does the \( x \)-coordinate of the vertex mean in this context?

The speed to travel the maximum distance

\[
d = -2(s-6)(s-6) + 72 \\
d = (-2s+12)(s-6) + 72 \\
d = -2s^2 + 24s + 12s - 72 + 72 \\
d = -2s^2 + 24s
\]

e) What is the \( y \)-intercept? What does it mean in this context?

0 is the \( y \)-intercept. Before you start the speed is 0.