Exponential Decay:

Exponential Relations: \( y = A(b)^t \)  \( A = \) \( \text{initial} \)  \( b = \) \( \text{base} \) \( \text{(multiplying factor)} \)

Decaying when \( 0 < b < 1 \)

Examples of Exponential Decay:

Half Life \( y = A \left( \frac{1}{2} \right)^t \) \( A \) is the initial amount of material, \( t \) is time, \( h \) is half life

Depreciation \( b = 1 - \% \text{ pop decline} \)

Half-Life:

1) The half-life of carbon-14 is 5730 years. The relation \( C = \left( \frac{1}{2} \right)^{\frac{t}{5730}} \) is used to calculate the concentration, \( C \), in parts per trillion, remaining \( t \) years after death. Determine the carbon-14 concentration, rounded to three decimal places, in

a) A 5730 year-old fossil

\[
C = 1 \left( \frac{0.5}{5730} \right)^{5730} \\
C = 0.5
\]

b) Why does your answer from part a make sense?

c) A 10,000 year old animal bone?

\[
C = 1 \left( \frac{0.5}{10000/5730} \right)^{5730} \\
C = 0.3
\]
Population:

2) The population of a village is decreasing by 7% per year. In 2006, the population was 19 800.

a) Write an exponential relation that models the population, $P$, over time $t$. Use $t = 0$ to represent the year 2006.

$$P = 19800 \left(0.93\right)^t$$

b) Use the relation from part a) to estimate the population in 2010.

$$P = 19800 \left(0.93\right)^4$$

$$P \approx 14811$$

2006 $\rightarrow$ 2010

c) In what year will the population decrease to half its 2006 value?

Guess & Check

$$19800 \left(0.93\right)^t \approx 9900$$

2015-2016

Practice:

1) Stage lights are often covered with gels to colour the light, but this also reduced the intensity of the light. The intensity of light, $I$, in watts per square centimetre is given by the relation $I = 1400 \left(\frac{1}{2}\right)^n$, where $n$ is the number of gels used. Find the intensity of light with 5 gels.